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Lumped Circuit Heat Model of Photonic Crystal Membrane Nanocavities

Thomas Van Vaerenbergh, *Student Member, IEEE*, Joni Dambre, *Member, IEEE*, Peter Bienstman, *Member, IEEE*, Juan Ariel Levenson, *Member, IEEE*, and Alejandro M. Yacomotti, *Member, IEEE*

***Abstract*— An InP L3 photonic crystal cavity on a suspended membrane has a characteristic time of ∼1 μs, which is 5.4× slower than the thermal cavity lifetime. To explain this mismatch, we use a lumped circuit model containing not only the cavity temperature, but also an average temperature of the membrane. Comparison of this model to a 1D COMSOL simulation, results in a thermal time constant of the membrane that is 7.9× slower than the thermal cavity lifetime, and slows down the transient response. The membrane behaves similar to other membrane cavity types.**

***Index Terms*— Lumped circuit, photonic crystal cavity, suspended membrane, thermal dynamics.**

I. Introduction

The tight light confinement in low-loss Photonic Crystal (PhC) nanocavities tremendously enhances the light-matter interaction, and hence paves the way to a variety of applications [1]–[4]. An important phenomenon here is linear absorption, which heats the cavity, and consequently changes its resonance wavelength due to the thermo-optic effect [5], [6]. In combination with free-carrier nonlinearities, this can give rise to bistability, self-pulsation and excitability [1]. Hence, an accurate model of the thermal dynamics is needed.

Whereas the thermal steady-state behavior is rather well-known, the dynamic behaviour is far from being trivial. Indeed, e.g., in [7], the transient response of the cavity corresponds to a thermal relaxation time $\tau_{th} \sim 1$ μs, which is 5.4× slower than 186 ns, the value one would expect based on the measured thermal resistance of the cavity and its heat capacity. The solution of the heat diffusion equation for an in-plane 2D description of the cavity confirms this difference between the transient relaxation time (∼300 ns) and the value predicted by the cavities thermal resistance and heat capacity (50 ns). In this letter, we will explain this apparent discrepancy using an intuitive lumped circuit model. Therefore, in addition to an averaged cavity temperature, we need to include an averaged temperature of the membrane in our model. This approach was previously proposed in the case of a GaAs-based L3 PhC cavity [8], where the incorporation of a membrane temperature in the thermal model explained the dynamics in a PhC cavity more accurately. We will illustrate that this thermal model can also be used for PhC membrane cavities in other material systems, such as the InP-based L3 cavity design proposed in [7] of which the geometry and physical parameters are shown in Fig. 1 and Table I, respectively. As in [7] we use effective parameters, that incorporate the effect of the holes in the membrane.

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T. Van Vaerenbergh and P. Bienstman are with the Photonics Research Group, Ghent University, Ghent B-9000, Belgium, and also with the Center for Nano- and Biophotonics, Ghent University, Ghent B-9000, Belgium (e-mail: thomas.vanvaerenbergh@intec.ugent.be; peter.bienstman@ugent.be).

J. Dambre is with the ELIS Department, Ghent University, Ghent B-9000, Belgium (e-mail: jdambre@elis.UGent.be).

J. A. Levenson and A. Yacomotti are with the Laboratoire de Photonique et de Nanostructures, Marcoussis 91460, France (e-mail: ariel.levenson@lpn.cnrs.fr; alejandro.giacomotti@lpn.cnrs.fr).

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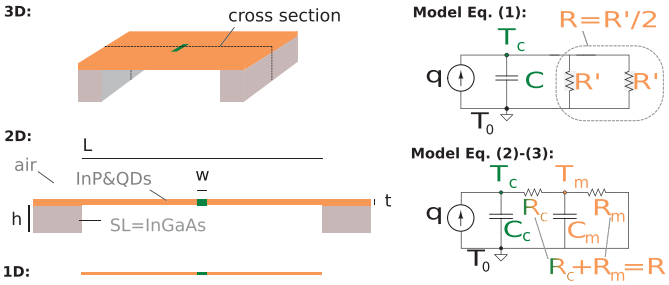

Fig. 1. A reduction of the geometry of a PhC cavity on a suspended membrane, to the principal dimension in which the heat flow takes place, gives rise to simple lumped circuit models. Incorporating the membrane temperature T_m in the model improves the description of the dynamics.

TABLE I
VALUES OF THE PARAMETERS USED IN THE SIMULATIONS [7], [9]

Parameter	Value	Magnitude
$\rho_m c_{p,m} = \rho_c c_{p,c}$	1.500	J/(gK)
$\rho_{GaInAs} c_{p,GaInAs}$	1.65	J/(gK)
κ_m	46	W/(mK)
κ_c	68	W/(mK)
κ_{GaInAs}	5	W/(mK)
w	450	nm
L	10	μm
t	250	nm
h	1.16	μm

II. Temperature Model

A. Single Temperature Variable

When describing thermal nonlinearities in cavities, the temperature dynamics are most often described using a lumped circuit approach, in which the average cavity temperature follows a differential equation [7], [10]–[13]:

$$\frac{dT_c}{dt} = -\frac{(T_c - T_0)}{RC} + \frac{\dot{q}}{C}. \quad (1)$$

In this equation, $\dot{q}$ is the heat source, T_c is the temperature of the cavity, T_0 is the temperature of the environment, R is the thermal resistance and C is the heat capacity of the system. Additionally, as long as R is well-chosen Eq. (1) results in the correct equilibrium temperature $T_{eq} = T_0 + R\dot{q}$.

The thermal resistance R of the PhC cavity in Fig. 1 is mainly determined by the direction in which the distance between the cavity and the membrane edge is shortest [7]. So, we can use a cross-section in this direction, orthogonal to the cavity plane, to model the heating process. In that cross-section $L = 10$ μm is the width of

the membrane in that direction, and $w = 450$ nm the width of the cavity, which is centered symmetrical in the middle of the membrane. If we work in a 1D model (see Fig. 1, 1D-sketch) and assume the temperature at the edges of the membrane to be T_0, the thermal resistances of the membrane parts at each side of the cavity are connected in parallel, such that $R = \frac{R'}{2} \approx \frac{1}{2}\frac{L/2}{\kappa_m A}$, with R' the thermal resistance of half of the membrane, κ_m the thermal conductivity of the membrane and A the cross-section of the cavity orthogonal to the direction of the heat flow (which will not influence the timescale). In other photonic cavities one often takes $C = C_c$, with C_c the heat capacity of the cavity [11]–[13]. But, as $C_c = \rho_c c_{p,c} A w$, with ρ_c representing the density of the cavity material and $c_{p,c}$ the corresponding specific heat capacity, this results in a thermal lifetime $\tau = RC_c \approx 37$ ns, while the experimental timescale is $\sim 1\,\mu s$ [7]. Furthermore, there is also a mismatch between this estimate of the thermal cavity lifetime, which is $\propto L/2 \times w/2$, and the timescale one obtains using a first order approximation of the heat diffusion equation, which is $\propto (L/2)^2$ [7].

B. Two Coupled Temperature Variables

To solve the mismatch between the prediction of τ_{th} based on Eq. (1) and the experimental results, without having to solve the full heat diffusion equation, we can, using the approach hinted at in [8], extend the previous model to:

$$\frac{dT_c}{dt} = -\frac{(T_c - T_m)}{R_c C_c} + \frac{\dot{q}}{C_c}, \tag{2}$$

$$\frac{dT_m}{dt} = -\frac{(T_m - T_0)}{R_m C_m} + \frac{(T_c - T_m)}{R_c C_m}. \tag{3}$$

Here, the first equation describes heat dissipation from the cavity to the membrane and the second equation describes heat dissipation from the membrane to the surroundings. Whereas in many other photonic cavities this second equation can be safely ignored [6], [10]–[12], we will demonstrate in this letter that, similar to [8], this is not the case for the InP PhC cavity. In Eq. (2)–(3), T_c is still the temperature of the cavity, but we also include the characteristic temperature of the membrane T_m. Hence, R_c is now the thermal resistance between the cavity and the positions on the membrane with temperature T_m, and R_m is the thermal resistance between these positions and the membrane edge (which we consider to be at a temperature T_0). Furthermore, we also include the influence of the heat capacitance of the membrane $C_m = \rho_m c_{p,m} A(L - w) \approx \rho_m c_{p,m} AL$ (as $w \ll L$) in our model, which was ignored in Eq. (1). As long as $R = R_c + R_m$, the thermal series resistance of R_c and R_m equals R, and consequently Eq. (2)–(3) will result in the same steady-state temperature of T_c as in (1).

If we use the horizontal symmetry of the structure and define $fL/2$ to be the distance between the edge of the membrane and the point with temperature T_m, we obtain $R_c \approx \frac{1}{2}(1-f)\frac{L/2}{\kappa_m A}$ (as $w << L$ we neglect the influence of $\kappa_c \neq \kappa_m$) and $R_m = \frac{1}{2}(f)\frac{L/2}{\kappa_m A}$. This position fraction $f \in [0, 1]$ can be used as a fitting value of this model. If $f = 0$, the model reduces to Eq. (1), while if $f = 1$, the cavity temperature instantaneously follows the membrane temperature which has its own intrinsic timescale

$$\tau_m = R_m C_m \approx f\frac{\rho_m c_{p,m}}{\kappa_m}(L/2)^2. \tag{4}$$

This is the slowest timescale in this system, and is $\propto (L/2)^2$. Intuitively, it can be understood that this slower timescale allows the model to correspond more closely to the result of the heat diffusion equation. Indeed, if $f = \frac{4}{\pi^2}$, we get $\tau_m \approx \frac{4}{\pi^2}\frac{\rho_m c_{p,m}}{\kappa_m}(L/2)^2 \approx 330$ ns, the timescale that corresponds to the lowest order term of the Fourier series of the solution of the heat diffusion equation [7].

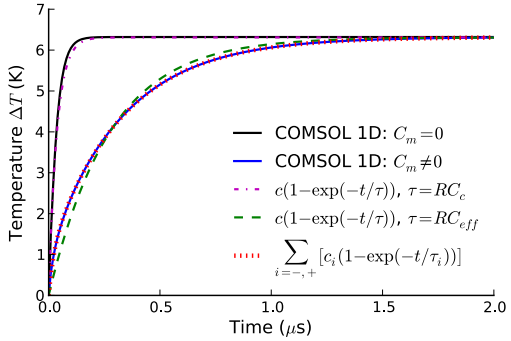

Fig. 2. Neglecting C_m (black solid line) results in an underestimate of the timescale of the heat response of a 1D model of a PhC cavity (blue solid line). An extended lumped circuit model (red dotted line) explains this thermal response better than the version that is often assumed in literature (magenta dash-dotted line). This is due to the incorporation of T_m in the extended model, explaining why the simpler model can only approximate the real response if an effective heat capacity C_{eff} is assumed (green dashed line).

III. Results

A. Comparison With 1D COMSOL Simulation

If we heat the cavity with a step function the solution for ΔT_c of Eq. (1) can be written as one rising exponential $c(1 - \exp(-t/\tau))$, while the solution of Eq. (2)–(3) can be written as a sum of two rising exponentials: $\sum_{i=-,+}[c_i(1 - \exp(-t/\tau_i))]$. The fast ($\tau_-$) and slow ($\tau_+$) timescale of this thermal response can be calculated from the physical timescales $\tau_c = R_c C_c$ and $\tau_m = R_m C_m$ ($k = \frac{\tau_m}{\tau_c}$ and $g = \frac{R_m}{R_c}$):

$$\frac{\tau_{-,+}}{\tau_c} = \frac{2k}{1 + k + g \pm \sqrt{(k-1)^2 + 2\,k\,g + 2\,g + g^2}}. \tag{5}$$

Consequently, $\tau_- \leq \tau_c$ and $\tau_+ \geq \tau_m$. Furthermore, using the previous approximations for the resistances and capacities, we can calculate an initial guess for $\tau_- = 21$ ns and $\tau_+ = 350$ ns. In Fig. 2, we compare a 1D COMSOL simulation of the system (blue solid line), with a simulation in which we assumed $c_{p,m} = 0$, hence $C_m = 0$ (black solid line). The heat capacity of the membrane clearly influences the heating dynamics of the cavity, neglecting this factor results in a serious underestimate of τ_{th}. Hence, using Eq. (1) in combination with $C = C_c$ (magenta dash-dotted line), results in an approximation of the response for $C_m = 0$ instead of $C_m \neq 0$.

Additionally, we fit the model proposed in Eq. (1), with a non-fixed value for C, and the model Eq. (2)–(3) (green and red dashed line, resp.) to the 1D COMSOL simulation for $C_m \neq 0$. The model of Eq. (2)–(3) is still more accurate than the model of Eq. (1), even though C is now variable. Furthermore, it gives a physical explanation why $C = C_{eff} > C_c$ in this simpler model: C_{eff} should also incorporate the influence of C_m. The fit of a rising exponential to the COMSOL time trace results in $\tau = 274$ ns, while the better fit of the extended model results in $\tau_c = 31.2 \pm 0.1$ ns, $\tau_m = 245 \pm 3$ ns $\approx 7.9\tau_c$ and $\frac{R_m}{R_c} = 2.41 \pm 0.01$. The latter corresponds to $\tau_- = 23$ ns and $\tau_+ = 329$ ns. The difference with our initial prediction is not only due to the crude approximations we made, but is also inherent to the lumped circuit approximation we use: as the conductivities and capacities of the cavity and membrane are in the same order of magnitude, the boundaries between cavity and membrane are rather ill-defined. As the heat capacities scale with the volumes of the cavity and membrane, this implies that the fit delivers effective capacity values for C_c and C_m, that are not purely geometrically defined.

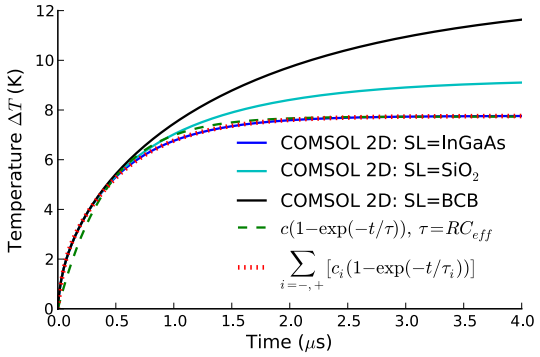

Fig. 3. The extended lumped circuit model (red dotted line) still explains the 2D simulation of the heat response of a PhC cavity (blue solid line) better than the fit of a single rising exponential (green dashed line). The choice for InGaAs in the sacrificial layer (SL) instead of, e.g., SiO_2 (solid cyan line) or BCB (solic black line), clearly influences the dynamics. The parameters used in this simulation can be found in Table I.

B. Comparison With 2D COMSOL Simulation

Using the same model we can fit τ_c and τ_m to a 2D simulation of the heat diffusion in the plane orthogonal to the cavity plane (see Fig. 1, 2D-sketch), in the direction of the shortest distance between the cavity and the membrane edge (Fig. 3). We obtain $\tau_c = 58 \pm 1$ ns, $\tau_m = 424 \pm 10$ ns. and $\frac{R_m}{R_c} = 1.90 \pm 0.02$. The main difference between the 2D and 1D simulation is the incorporation in the simulation of the GaInAs sacrificial layer under the InP (SL, Fig. 1). Consequently, this layer with low conductivity clearly slows down the heat diffusion of the cavity and, as discussed in [7], the choice of sacrificial layer material influences the relaxation time (comparison with BCB and SiO_2 included in Fig. 3). The difference between the fit of the model and the COMSOL 2D simulation can also be attributed to the presence of this sacrificial layer in the simulation, as it is not explicitly incorporated in the model. A full 3D-finite element numerical simulations, as in Ref. [14], would account even better for the thermal dynamics in this system.

C. Optically Coupled Cavities

Finally, this lumped circuit model can be used to predict complex dynamics in PhC coupled cavity systems. The typical intercavity distance needed for optical coupling, $d \sim 1$ μm, is at least $\sim 3\times$ smaller than the distance from the cavity to the edge of the membrane $\sim L/2$ [3]. Combined with the important role of the heat conduction through the membrane, we expect that optically coupled cavities operating in the high power regime will suffer from severe thermal coupling. Consequently, a good lumped circuit model of this system should definitely incorporate the thermal heat flow between the cavities.

IV. Conclusion

In conclusion, when describing the heat dynamics in a membrane PhC cavity in a lumped circuit model, the heating of the membrane needs to be incorporated in the model, otherwise the thermal lifetime is underestimated. Additionally, the choice of sacrificial layer influences the thermal dynamic response.

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