Self-localized Waveguides in Nonlinear Photonic Crystals

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Abstract: We study a new variety of self-localized Bloch modes or gap solitons in Kerr nonlinear high-contrast photonic crystals without defects. In addition to rigorous calculations, we develop a semi-analytical approach using a folded Green's function.

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1. Introduction

The combination of photonic crystals and nonlinearities is the subject of intense research. Because of the strong dispersion and localization effects in bandgap structures, the weak nonlinearities can be exploited. The use of high-Q small volume resonators e.g. provides extremely compact all-optical switching and add-drop functionalities. On the other hand, the inclusion of the Kerr effect gives rise to nonlinear energy localization. In combination with the discrete nature of the photonic crystals this may lead to nonlinear modes. These are called gap solitons or discrete breathers.

In this paper we examine a novel kind of solitary wave. The used geometry is a photonic crystal without defects. Thus, in the linear case a wave with frequency in the bandgap is exponentially dampened. However, in the nonlinear regime, the wave creates its own waveguide channel, by changing the indices of the center rods. In the transversal direction the mode is still confined because of the bandgap. We found both on-site and inter-site solitons. Using energy arguments and simulations we determined the on-site variety to be stable.

Furthermore, we developed a semi-analytical theory using the so-called Strip-Green's function. This is a confined version of the normal Green's function, with application of Bloch boundary conditions. With approximation, the resulting integral equation is transformed into a system of nonlinear algebraic equations. The coefficients describe the long-range nonlinear effective discrete interactions.

In addition, the previous concepts have been applied to a reduced symmetry photonic crystal. In this system, consisting of large linear rods and small nonlinear rods, we can also find self-localized waveguides.

2. Rigorous modeling method

We developed an extension to the linear eigenmode expansion method for the Kerr effect [1-3]. In this algorithm the nonlinear parts of the two-dimensional structure are divided in a grid. An iteration of linear eigenmode calculations is performed, each time updating the refractive index profile. It can be used for both finite and periodic infinite structures. In the latter case we calculate the Bloch modes at each step and we keep the flux constant to keep the iterations self-consistent. Because the unit cells are often small, and the needed number of eigenmodes for accurate results is limited, the calculations are efficient. Furthermore, the simulation can start from a previously obtained approximate solution. This leads to calculation times of seconds or minutes on a modest workstation.

3. Self-localized waveguides

The used photonic crystal is a square lattice of squares with index 3.4 in an air background. We know that lowering the index of a line of defects may lead to a waveguide, therefore we assume a negative Kerr nonlinearity. As a 'seed' for the simulation we use such a waveguide mode. If the nonlinear iteration converges we have obtained a self-localized waveguide, that carves its own channel. We indeed find such modes, and the on-site type is depicted in Figure 1. The on-site variety has a maximum in the center, whereas the inter-site variety has a node in the center.

An important issue with these kinds of modes is their stability. Here, we can apply Peierls-Nabarro potential ideas. According to this theory, two solitary waves with the same topology are two variants of the same entity. Then, the one with lowest energy will be stable, whereas the other one will be unstable. If they have the same energy the mode is expected to be mobile over the lattice, switching without potential barrier between the two types.

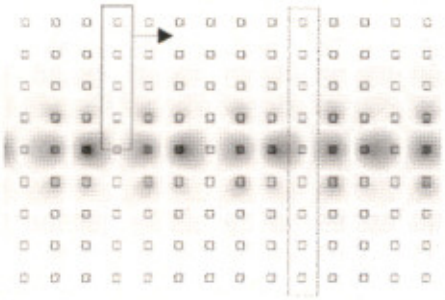

Fig. 1. Depiction of an on-site self-localized waveguide. The small box is the Bloch calculation area.

The theory can be used for our inter-site and on-site modes, because they are both staggered. We performed the energy calculations and the results are shown in Figure 2. We see that for frequencies near the low bandgap edge, the energies coincide, therefore these modes are expected to be mobile in the transverse direction, if they are excited with some transverse 'momentum'. For higher frequencies, the on-site variety has the lowest energy, and is thus stable. This is in agreement with numerical experiments on the large box in Figure 1. If we perturb the inter-site type slightly, the iteration converges to an on-site mode. Whereas, the on-site modes are quite robust.

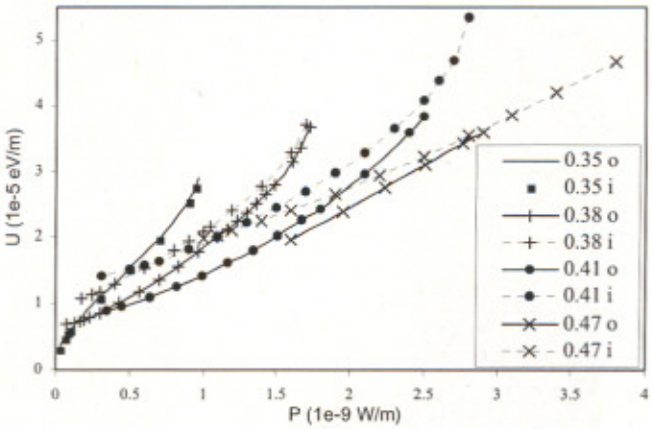

Fig. 2. Energy U versus flux P of the modes. The legend shows a/λ with 'o'=on-site and 'i'=inter-site.

4. Semi-analytical theory

To gain insight we set out to find a more theoretical description. To this end we combined two concepts. The Green's function is an efficient tool to model linear and nonlinear photonic crystal modes. Furthermore, a variant, the Strip Green's function, is defined for the description of linear waveguide modes [4]. Next, for nonlinear interactions, effective discrete equations have been derived for the whole space Green's function [5]. Our problem is especially suited for the Strip Green's function, and we have combined it with the effective equation formalism.

We restrict the perturbed Helmholtz problem to a strip (x,y)=[-a/2,a/2]x3, with x the propagation direction and y the transversal direction:

$$\left[\nabla^2 + \varepsilon_{pc}\left(\frac{\omega}{c}\right)^2\right] E(x,y \mid \omega,k_x) = -\varepsilon_d\left(\frac{\omega}{c}\right)^2 E(x,y \mid \omega,k_x)$$

with the Bloch condition $E(x+a,y \mid \omega,k_x) = E(x,y \mid \omega,k_x)e^{ik_x a}$. Here, a is the period of the photonic crystal. The index is split in two parts $\varepsilon = \varepsilon_{pc} + \varepsilon_d$ with the first part presenting the perfect linear photonic crystal, and the second part indicating the defects, in our case the defect is the Kerr nonlinearity:

$$\varepsilon_d = -\delta_{rods}(x,y) \mid E \mid^2$$

with δ_{rods} zero outside the rods and one inside. For the homogeneous Helmholtz problem in the strip, a Green's function g can be constructed [4], the relation with the whole space Green's function G is a kind of folding procedure:

$$g(\bar{r},\bar{u}\mid\omega,k_x)=\sum_{j\in Z}G(\bar{r},\bar{u}\mid\omega,k_x)e^{ijk_x a}\,.$$

The Green's formalism leads to an integral equation. If we consider the field to be constant in a rod, we can simplify to a set of discrete equations for the rod fields E_n [5]:

$$E_n=\sum_m J_{n-m}\mid E_m\mid^2 E_m,$$

with the interaction coefficients J_{n-m} defined as an integral of g over a rod:

$$J_n=-\left(\frac{\omega}{c}\right)^2\int_{rod}g(\bar{r}_0,\bar{r}_n+\bar{u}\mid\omega,k_x)d^2\bar{u}\,.$$

In the photonic crystal of the previous paragraph, the agreement with the rigorous calculations is qualitative, however a larger parameter space can quickly be covered, see Fig. 3. We notice the difference occurs mainly in the center rod n=1.

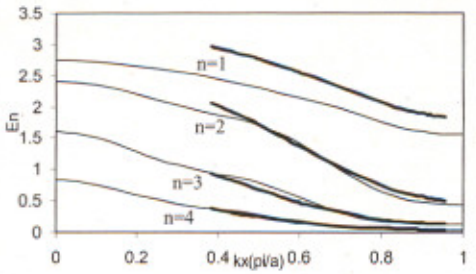

Fig. 3. Comparison between rigorous (thick line) and strip Green calculations (thin line) of the rod fields E_n.

5. Reduced symmetry photonic crystal

The reduced symmetry or diatomic photonic crystal consists of two offset lattices (see inset Fig.4), one with large linear rods (radius=0.2a) and one with small nonlinear rods (r=0.1a). Here too, we find gap solitons, but with positive nonlinearity. Because of the small Kerr rods, the agreement with the theory becomes quantitive, further validating the approach.

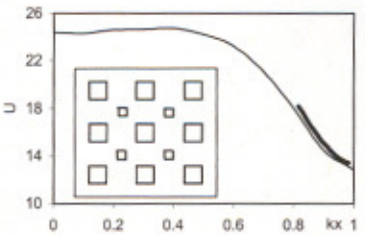

Fig.4. Comparison of energy versus k_x for a diatomic photonic crystal. Inset shows the geometry.

6. Conclusions

We have presented and studied novel kinds of self-localized waveguides with both rigorous calculations and a semi-analytical strip Green theory.

7. References

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