Optical phase shifting with 1D Kerr-nonlinear resonators

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Kerr-nonlinear resonators show great potential both for phase shifting and optical bistability/limiting and may therefore improve the functionality of future optical communication systems. Here we explain for the first time the principle of phase shifting analytically in the 1D case. Excellent agreement with numerical simulations is obtained. We demonstrate optimal design parameters for several signal bandwidths (2, 10, 40 and 100 GHz) and show that a trade-off between signal bandwidth, input intensity and device length is to be made.

Introduction

The usability of the ultrafast optical Kerr effect in all-optical processing has always been hindered by the need for high input power or long devices, because of the small value of n_2 in semiconductors (order $10^{-13} cm^2/W$). Periodic structures made of resonators however confine the optical power and slow down the propagation of the pulse. This reduces the input power or device length needed to achieve a certain phase shift [1, 2]. In addition, they give rise to new effects like optical bistability [3] and optical limiting [4], which can not be observed in simple homogeneous structures.

However the action of a resonating structure is not simply boosting the power and slowing down the pulse. What typically happens, is that its transmission window shifts and, with it, the window widens or narrows. It can roughly be said that the resonance shift is determined by the overall value of n_2, while the change of width is due to the modulation of n_2. From here on, we assume the value of n_2 to be constant throughout the resonator (the underlying idea is to model a weakly corrugated waveguide by a 1D approach).

Linear properties of the resonator structure

The resonator we use, has the following period,

$$h_{\frac{\lambda_c}{8}}\ l_{\frac{\lambda_c}{4}} h_{\frac{\lambda_c}{4}} l_{\frac{\lambda_c}{4}} \ldots l_{\frac{\lambda_c}{4}} h_{cav.\frac{\lambda_c}{2}}\ l_{\frac{\lambda_c}{4}} h_{\frac{\lambda_c}{4}} l_{\frac{\lambda_c}{4}} \ldots l_{\frac{\lambda_c}{4}} h_{\frac{\lambda_c}{8}}$$

with h resp. l indicating the higher resp. lower index material, cav a integer number and λ_c a chosen resonance wavelength. We easily recognize the mirror and cavity part. Any multiple of this structure has a transmission of 1 for $\lambda = \lambda_c$. In general, it will take more than one resonator to achieve a certain phase shift.

The dispersion relationship for an infinite number of resonators in the transmission window around the central wavelength λ_c is in good approximation,

$$\nu - \nu_c = \mp \frac{\Delta\nu}{2} \sin(kL \pm \frac{\pi}{2}) \tag{1}$$

with $\Delta\nu$ the resonance bandwidth. The exact sign depends on the number of lower index layers in one mirror being even or odd. We can rewrite this relation as,

$$\phi_L \pm \frac{\pi}{2} = \mp \arcsin(\frac{2}{\Delta\nu}(\nu-\nu_c)) = \mp \arcsin(\left|\frac{d\phi_L}{d\nu}\right|_{\nu_c}(\nu-\nu_c)) \tag{2}$$

where $\phi_L \equiv kL$ is the linear phase change between input and output of a single period (period length L). On the other hand, the amplitude transmission and phase relation around λ_c for a single, isolated resonator is approximately given by,

$$|t|_L = \frac{1}{\sqrt{1+\left|\frac{d\phi_L}{d\nu}\right|^2_{\nu_c}(\nu-\nu_c)^2}} \tag{3}$$

$$\phi_L \pm \frac{\pi}{2} = \mp \arctan(\left|\frac{d\phi_L}{d\nu}\right|_{\nu_c}(\nu-\nu_c)) \tag{4}$$

Both situations are exactly the same for $\lambda = \lambda_c$, but deviate substantially when approaching the boundaries of the transmission window.

Kerr-nonlinear properties

As mentioned above, the nonlinear action of the Kerr effect will shift the transmission window, which will give rise to a shift of the phase. It is clear that this phase shift will increase both for larger ν_c-shift (or λ_c-shift) and higher $\left|\frac{d\phi_L}{d\nu}\right|_{\nu_c}$. It can be shown that the resonance shift $\Delta\nu_c$ is approximately given by

$$\Delta\nu_c = -\frac{n_h}{\frac{3}{4}n_2\frac{1+|r|}{1-|r|}|E_{in}|^2}\nu_c \cdot \frac{cav+7.64}{cav+4.19} \cdot \frac{3.03\frac{\Delta n}{n}+0.52}{4.31\frac{\Delta n}{n}+0.35} \tag{5}$$

with E_{in} the input electric field of the resonating structure, r the amplitude reflectivity of a single mirror, $\Delta n \equiv n_h - n_l$ and $n \equiv \frac{n_h+n_l}{2}$. The first part of equation (5) corresponds to the expected shift when the cavity would be large compared to the mirrors. The formula can be derived using a multiple time-scale approach with only the cavity taken into account. The *cav* and $\frac{\Delta n}{n}$ factor were derived from numerical simulations. However, their existence can be easily understood. The reflectivity amplitude $|r|$ of a mirror is almost constant with respect to the wavelength. However its phase can change significantly, certainly for longer mirrors characterized by a lower index contrast. Therefore the Kerr-nonlinear phase change in the cavity (leading to a new resonance frequency because of the round trip phase condition of a resonator) partially serves to compensate this change in the mirrors at the new resonance frequency and cannot fully be used for altering the refractive index in the round trip condition. We obtained a RMS error of 0.5% with this formula for a large variety of parameters (Δn, n, *cav*, E_{in}, n_2 and r). A formula for $\left|\frac{d\phi_L}{d\nu}\right|_{\nu_c}$ can be derived analytically.

We can now predict the phase shift over 1 period. In the case of an infinite number of resonators we have,

$$\phi_{NL} \pm \frac{\pi}{2} = \mp \arcsin(\left|\frac{d\phi_L}{d\nu}\right|_{\nu_c}(\nu-\nu_c-\Delta\nu_c)) \tag{6}$$

since the resonance bandwidth does not change (see introduction). This leads to the following shift at $\nu = \nu_c + \frac{\Delta\nu_c}{2}$ (center frequency between the linear and nonlinear resonance frequency),

$$|\Delta\phi| \quad = \quad 2\arcsin(\left|\frac{d\phi_L}{d\nu}\right|_{\nu_c} \frac{\Delta\nu_c}{2}) \tag{7}$$

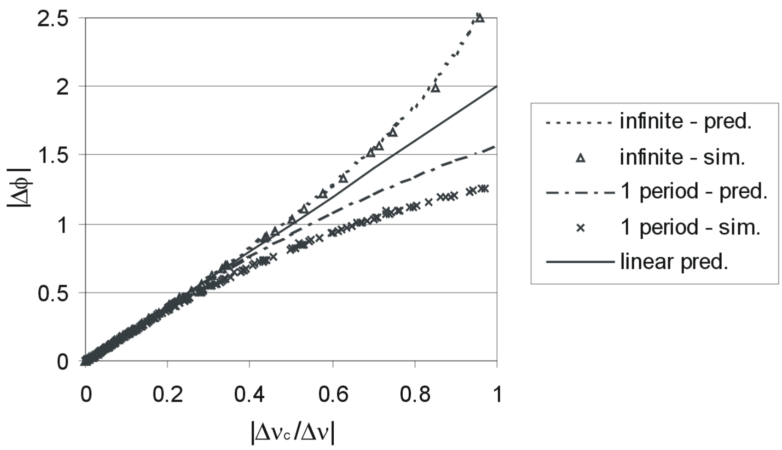

Figure 1: Predicted and simulated $|\Delta\phi|$ versus $\frac{\Delta\nu_c}{\Delta\nu}$ for a variety of parameters

In Fig. 1, the phase shift $|\Delta\phi|$ over 1 period is plotted versus the relative frequency shift $\frac{\Delta\nu_c}{\Delta\nu}$. A relative frequency shift > 1 is clearly not meaningful for phase shifting properties since no transmission overlap between linear and nonlinear regime is left. Excellent agreement is found between (7) and simulation results in the case of an infinite number of resonators. In the same way, we would naively expect that in the case of a single resonator,

$$|\Delta\phi| \quad = \quad 2\arctan(\left|\frac{d\phi_L}{d\nu}\right|_{\nu_c} \frac{\Delta\nu_c}{2}) \tag{8}$$

This is however only correct for small $\Delta\nu_c$: the nonlinear effect will not shift the transmission relationship homogeneously in the case of 1 period (only one point has $|t(\nu)|^2 = 1$). The shift will instead be proportional to $|t(\nu)|^2$ resulting in an asymmetric lorentzian transmission relation and the phase relation will be distorted in the same way. So the formula (8) will only be correct if $\left|t(\nu_c + \frac{\Delta\nu_c}{2})\right|^2 \approx 1$, which can also be seen in Fig. 1. From this, we can conclude that in the general case of a finite number of resonators,

$$|\Delta\phi| \quad \approx \quad \left|\frac{d\phi_L}{d\nu}\right|_{\nu_c} \Delta\nu_c \tag{9}$$

which is in agreement with intuitive reasoning.

Design of π-phase shifting device

To be of use in practical devices, a phase shift of π should be realizable with $E_{in} < 200kV/cm$, total device length $L_{tot} < 1mm$ and reasonable signal bandwidth (overlap of linear and nonlinear bandwidth, taken into account that only the central half of the transmission window is really usable). However a trade-off must be made since a large signal bandwidth and small E_{in} conflict with a large $\Delta\phi$ and thus a small L_{tot}. This is shown in Fig. 2 for a realistic example: $n_h = 2.6$, $n_l = 2.36$ (thus a corrugation of $\approx 10\%$ and $n_2 = 0.6 \times 10^{-13} cm^2/W$ (or $2.4 \times 10^{-16} cm^2/V^2$). The number of mirror layers, the cavity length and the number of resonators are optimized with respect to the total device length by means of the analytical results presented above.

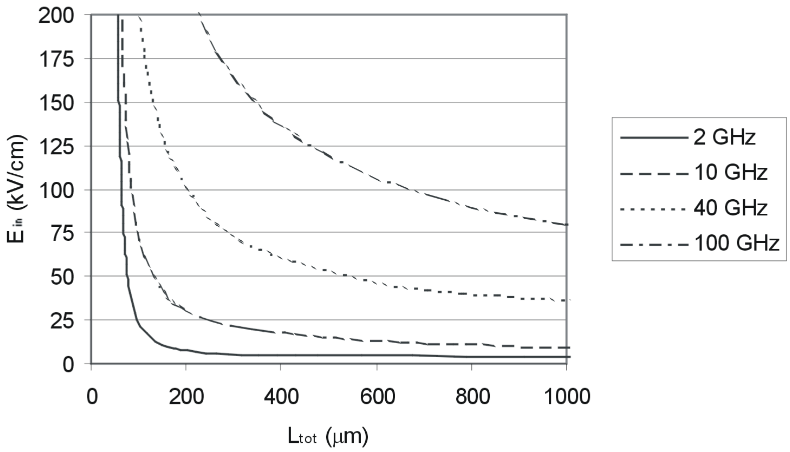

Figure 2: Trade-off between input field and device length for several signal bandwidths.

Note that without resonators, the device length needed to achieve a phase shift of π would be about $8cm$ for $E_{in} = 200kV/cm$. This means that resonators give rise to tremendous improvements for the purpose of phase shifting, however partially limited by the required signal bandwidth.

Conclusion

The influence of the nonlinear Kerr effect on resonator structures was explained analytically and excellent agreement with simulation results was obtained. Resonators highly reduce the required input power or device length when used for all-optical phase shifting. A phase shift of π is achievable, however the requirements increase rapidly with the signal bandwidth and a trade-off must be made.

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